The Relativity Mission, Gravity Probe B
Results and Lessons Learned
Sasha Buchman for the GP-B team

Q2C, Quantum to Cosmos #5
Köln, October 9th 2012
1. Experiment description

2. Patch effects

3. Data models and analysis techniques

4. Results, uncertainties and cross-checks

5. Lessons learned
Gravity Probe B Concept

\[ \vec{\Omega}_{FD} = \frac{G \cdot I_e}{c^2 R^3} \left[ \frac{3\vec{R}}{R^2} \vec{\phi}_e \cdot \vec{R} - \vec{\omega}_e \right] \]

Guide STAR
IM Pegasi
HR 8703

\[ \vec{\Omega}_G = \frac{3}{2} \frac{G \cdot M_e}{c^2 \cdot R^3} \vec{R} \times \vec{v} \]
Expected Gyroscope Behavior

*includes solar GR effects and guide star motion
Nothing could be simpler than GP-B; just a gyroscope and a telescope

*William Fairbank*
GP-B Gyroscope Requirements

1 marcsec/yr = 3.2 \times 10^{-11} \text{ deg/hr} = 1.5 \times 10^{-16} \text{ rad/sec}

- Electrostatic gyro uncompensated (10^{-1} \text{ deg/hr})
- Electrostatic gyro with modeling (10^{-5} \text{ deg/hr})
- Spacecraft gyros (3 \times 10^{-3} \text{ deg/hr})
- Laser gyro (10^{-3} \text{ deg/hr})

- 6,606 Geodetic effect
- 39 Frame dragging effect
- 0.50 GP-B requirement $\sim 10^{-6}$
Ensuring Gyro Performance

1. Optimize Geometry
   I. Gyro asphericity \( \delta R/R < 10^{-6} \)
   II. Gyro mass unbalance \( d_{MU}/R < 10^{-6} \)
   III. Housing asphericity \( \delta R_H/R_H < 10^{-5} \)

2. Reduce Environmental Disturbances
   I. Residual acceleration \( a_{trans} < 10^{-11} \text{ ms}^{-2} \)
   II. Magnetic field \( B < 10^{-10} \text{ T} \)
   III. Gas pressure \( P < 10^{-11} \text{ Pa} \)
   IV. Electric charge \( Q < 10^{-11} \text{ F} \)
   V. Patch effects \( \langle d_{\text{patch}} \rangle < 10^{-6} \text{ m} \)

3. Shift Frequency and Average Signal
   I. Roll satellite \( 77.5 \text{ s} \)
   II. Use polhode averaging \( \sim 3,000 \text{ s} \)
   III. Multiple gyroscopes (4) \( 4 \text{ gyros} \)

4. Use Natural Calibrations
   I. Daily aberration \( \sim 5 \text{ arcsec} \)
   II. Yearly aberration \( \sim 20 \text{ arcsec} \)

5. Ground Testing Capability
   \( 10 \text{ ms}^{-2} \text{ to } 10^{-11} \text{ ms}^{-2} \)
Gyro Description

- **Components**
  - Rotor
  - Housing
  - Read-out loop
  - Spin-up nozzle
  - UV fixtures
  - Suspension cables
  - Read-out cable

- **Functionality**
  - Electrostatic suspension
  - Capacitance rotor position
  - Forcing charge measurement
  - London moment read-out
  - Helium spin-up
  - Cryogenic operations
  - UV charge management

Gap = 32.5 \( \mu \text{m} \)
**Gyro Spin-up**

**Differential Pumping Requirement:**
1) spin channel ~ 10 torr (sonic velocity)
2) electrode area < $10^{-3}$ torr

<table>
<thead>
<tr>
<th>Gyro</th>
<th>$f$ (Hz)</th>
<th>$\frac{df}{dt}$ ($\mu$Hz/hr)</th>
<th>$\tau$ (yr)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>79.4</td>
<td>0.57</td>
<td>15,900</td>
</tr>
<tr>
<td>2</td>
<td>61.8</td>
<td>0.52</td>
<td>13,600</td>
</tr>
<tr>
<td>3</td>
<td>82.1</td>
<td>1.30</td>
<td>7,200</td>
</tr>
<tr>
<td>4</td>
<td>64.8</td>
<td>0.28</td>
<td>26,400</td>
</tr>
</tbody>
</table>

Gyro 1 spun-up last; spin-up of one gyro causes spin down of the other gyros.
London Moment Read-Out

- London moment read-out with dc SQUIDs
- Superconducting pickup on gyroscope housing
- \( < 8 \times 10^{-29} \text{ J/Hz} \) (<50 \( \mu \Phi_0 / \sqrt{\text{Hz}} \)) at 5 mHz \( \Rightarrow 200 \text{ marcsec/} \sqrt{\text{Hz}} (5 \times 10^{-11} \text{ G/} \sqrt{\text{Hz}}) \)

A spinning superconductor develops a magnetic “pointer” aligned with its spin axis

\[
B_L = -\frac{2mc}{e} \omega_s = -1.14 \times 10^{-7} \omega_s \text{ (G)}
\]
Gyroscope Readout Single Orbit

Output of SQUID Readout Electronics, Gyroscope 3, Orbit 6200, June 15, 2005

Optical Aberration due to Orbital Motion
Arc Sec
Time (minutes) After Acquisition of Guide Star

Output (Volts)

Time (minutes) After Acquisition of Guide Star

Arc Sec
Post Science Calibration Anomaly

Magnitude of Drift Rate vs. Angle of Misalignment

- Gyro 1
- Gyro 2
- Gyro 3
- Gyro 4

Drift Rate (as/day) vs. Angle of Misalignment (degrees)
The Patch Effect

Explanation of anomalies: **Patch Effect**
- Mechanical, magnetic, suspension effects ruled out
  - Variation of electric potential over the surface
    - *Can arise due to the polycrystalline structure*
    - *Can be affected by presence of contaminants*
  - Patch fields present on gyro and housing walls
  - Cause forces and torques between surfaces

**The Patch Effects:**
1. Gyro acceleration along the roll axis
2. Gyro acceleration at spin frequency
3. Gyro spin down
4. **Gyro polhode damping**
5. Misalignment torque
6. Resonance torque
7. Charge measurement bias

Schematic of surfaces with patches:
- Gyro surface
  - d
- Housing surface

Complicate Data analysis
3 Complications due to Patch Effect I

Misalignment Torques

- Proportional to spin to roll angle
- Magnitude up to 2.5 arcsec/yr
- Orthogonal to misalignment plane (roll & spin axes)

\[ k = 2.5 \, \text{arc sec/day/degree} \]
Polhode damping

- Caused by power dissipation in housing resistances to ground
- Changes trapped flux orientation to the spin axis
- Complicates scale factor $C_g$ determination
3 Complications due to Patch Effect III

Roll-polhode Resonance Torque

- ‘Jumps’ occur when harmonic of polhode rate coincident with roll rate
- Up to 200 marcsec discontinuities in data
- Long term oscillatory behavior

\[
\frac{m}{f_{roll}} = \frac{1}{f_{polh}}
\]

Gyro 2 per orbit orientation

\[s_{EW}\]

res. \(\text{m}\)

Date (2005):

May 146, 145, 144, 143
Jun 142, 141, 140
Jul 139, 138
Raw & Processed Flight Data (Gyro 2)

Gyro 2, orientations – Newtonian torques

EW orientation (arcsec)

NS orientation (arcsec)

NS uniform drift

EW uniform drift

date (2004)
Gyroscope Performance

1 marcsec/yr = 3.2 × 10^{-11} deg/hr = 1.5 × 10^{-16} rad/sec

- Electrostatic gyro uncompensated (10^{-1} deg/hr)
- Electrostatic gyro with modeling (10^{-5} deg/hr)
- Spacecraft gyros (3 × 10^{-3} deg/hr)
- Laser gyro (10^{-3} deg/hr)

- 6,606 Geodetic effect
- 700 marcs Patch effects
- 39 Frame dragging effect
- ~1% Data Analysis
- 0.50 GP-B requirement
Science Data Segments and Anomalies

- **Spinup & Alignment Complete Gyros 1, 2, 3**
- **Gyro 4**

**Timeline:**

- **Aug:** Segment 2
  - 1 - Gyro 3 Analog Backup
- **Sept:** Seg. 3
  - 2 - SRE Safemode
- **Oct:** Segment 5
  - 3 - bad GPS config
  - 4 - roll notch filter
  - 5 - Jan 20 Solar Flare
- **Nov:** Segment 6
  - 6, 7, 8 - computer reboots
  - 9 - roll notch filter
- **Dec:**
- **Jan:**
- **Feb:**
- **Mar:** Segment 9
- **Apr:**
- **May:** Segment 10
- **Jun:**
- **Jul:**
- **Aug:** Calibration
- **Sept:**
### Two Foundations of the Data Analysis

1. **Spectral separation**
   - a) Rotor spin \( \sim 60 \text{ Hz} - 80 \text{ Hz} \) *(changing with time)*
   - b) Spacecraft roll = 12.9 mHz *(from on-board star trackers)*
   - c) Spacecraft orbit = 0.17 mHz *(from on-board GPS)*
   - d) Rotor polhode \( \sim 0.1 \text{ mHz} \) *(changing with time)*
   - e) Earth’s orbit = 31.7 nHz *(from JPL Earth ephemeris)*

   ➔ **General Relativity acts at zero-frequency**

2. **Trapped magnetic flux** ➔ Enables determination of a) & d)
Sources of Experiment Uncertainty

1. **Statistical**
   - Covariance matrix (or Fisher info) provided by 2-sec Filter
   - Quantifies uncertainty related to:
     - measurement noise
     - a priori information
     - Model (including the number of parameters estimated)

2. **Systematic**
   A. Parameter sensitivity
      - Quantifies uncertainty associated with choice of number of parameters estimated
   B. Unmodeled effects
      - e.g. uncertainty of solar geodetic effect, guide star motion
# Mitigation of Systematic Effects

<table>
<thead>
<tr>
<th>Source</th>
<th>Magnitude of Effect</th>
<th>Analysis</th>
<th>Contribution to total error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Misalignment torque</td>
<td>~ 500 mas/yr</td>
<td>Physical Model</td>
<td>~5 mas/yr EW ~15 mas/yr NS</td>
</tr>
<tr>
<td>Resonance torque</td>
<td>~ 500 mas/yr</td>
<td>Physical Model</td>
<td>~4 mas/yr</td>
</tr>
<tr>
<td>Other classical torques</td>
<td>&lt; 1 mas/yr</td>
<td>Not modeled</td>
<td>&lt; 1 mas/yr</td>
</tr>
<tr>
<td>SRE variations</td>
<td>~ 100 mas/yr</td>
<td>Taylor expansion</td>
<td>~4 mas/yr EW ~12 mas/yr NS</td>
</tr>
<tr>
<td>Gyro scale factor</td>
<td>~ 100 mas/yr</td>
<td>Physical Model TFM</td>
<td>~1 mas/yr EW ~2 mas/yr NS</td>
</tr>
<tr>
<td>Other readout errors</td>
<td>&lt; 1 mas/yr</td>
<td>Not modeled</td>
<td>&lt; 2 mas/yr</td>
</tr>
<tr>
<td>Telescope readout</td>
<td>~ 1 mas/yr</td>
<td>Not modeled</td>
<td>~1 mas/yr</td>
</tr>
<tr>
<td>Guide Star Motion</td>
<td>~ 1 mas/yr</td>
<td>Not modeled</td>
<td>~1 mas/yr</td>
</tr>
<tr>
<td>ECU noise</td>
<td>10% data</td>
<td>Data excluded</td>
<td>~2 mas/yr</td>
</tr>
<tr>
<td>TOTAL</td>
<td>~700 mas/yr (GR ~6600 mas/yr)</td>
<td></td>
<td>~8 mas/yr EW ~20 mas/yr NS</td>
</tr>
</tbody>
</table>
## GP-B Results

### Relativistic Drift Rate Estimates

<table>
<thead>
<tr>
<th>Gyro</th>
<th>$r_{NS}$ (mas/yr)</th>
<th>$r_{WE}$ (mas/yr)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gyro 1</td>
<td>-6,588.6 ± 31.7</td>
<td>-41.3 ± 24.6</td>
</tr>
<tr>
<td>Gyro 2</td>
<td>-6,707.0 ± 64.1</td>
<td>-16.1 ± 29.7</td>
</tr>
<tr>
<td>Gyro 3</td>
<td>-6,610.5 ± 43.2</td>
<td>-25.0 ± 12.1</td>
</tr>
<tr>
<td>Gyro 4</td>
<td>-6,588.7 ± 33.2</td>
<td>-49.3 ± 11.4</td>
</tr>
<tr>
<td>Joint result</td>
<td><strong>-6,601.8 ± 18.3</strong></td>
<td><strong>-37.2 ± 7.2</strong></td>
</tr>
<tr>
<td>GR Prediction</td>
<td><strong>-6,606.1 ± 0.1</strong></td>
<td><strong>-39.2 ± 0.1</strong></td>
</tr>
</tbody>
</table>

### Contributions to Experiment Uncertainty

<table>
<thead>
<tr>
<th></th>
<th>$r_{NS}$ (mas/yr)</th>
<th>$r_{WE}$ (mas/yr)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Statistical uncertainty</td>
<td>16.8</td>
<td>5.9</td>
</tr>
<tr>
<td>Systematic uncertainty</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Parameter Sensitivity</td>
<td>7.1</td>
<td>4.0</td>
</tr>
<tr>
<td>Solar geodetic effect</td>
<td>0.3</td>
<td>0.6</td>
</tr>
<tr>
<td>Telescope readout</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>Other readout uncertainties</td>
<td>&lt;1</td>
<td>&lt;1</td>
</tr>
<tr>
<td>Other classical torques</td>
<td>&lt;0.3</td>
<td>&lt;0.4</td>
</tr>
<tr>
<td>Guide star proper motion uncertainty</td>
<td>0.1</td>
<td>0.1</td>
</tr>
<tr>
<td>Total uncertainty</td>
<td><strong>18.3</strong></td>
<td><strong>7.2</strong></td>
</tr>
</tbody>
</table>
Low & High Frequency SQUID Data

LF SQUID channel (780 Hz LP filter → 4 Hz LP filter → gain)
- 5 Hz continuous

HF SQUID channel (780 Hz LP filter)
- $10^6$ “snapshots” over 1 year
  (2200 Hz, ~ 2 sec duration)

Gyro 1 HF “snapshot”, 10 Nov. 2004
- Telescope measures $\hat{\tau}$
- SQUID measures $\vec{\mu} = \hat{\tau} - \hat{S}$

\[ Z = C_g \mu_{NS} \cos \phi_r + \delta \phi + \mu_{EW} \sin \phi_r + \delta \phi + b + n \]

- Gyros 2 & 1
- Gyros 4 & 3
- North (inertial)
- S/C x-axis
- Telescope
- Guide Star (IM Pegasi)
Time-varying Scale Factor, $C_g$

$$C_g = C_g^{SRE} C_g^{LM} + C_g^{TF}$$

- **Due to SQUID Readout**
  - Varies by up to $10^{-3}$
  - Modeled to < $10^{-4}$

- **Due to London Moment**
  - Constant to $\sim 10^{-5}$
  - Not modeled

- **Due to Trapped Flux & polhode motion**
  - Varies by up to $10^{-2}$
  - Modeled to < $10^{-4}$ by TFM

- **3 independent sources of evidence for SRE variations of up to $10^{-3}$**
  - Injected calibration signal
  - Flux slipping
  - Aberration of starlight

- **Inclusion of SRE model improves segment-to-segment consistency**

$$Z = C_g \mu_{NS} \cos \phi_r + \delta \phi + \mu_{EW} \sin \phi_r + \delta \phi + b + n$$
Gyroscope Equations of Motion (NS, EW)

\[
\frac{ds_{NS}}{dt} = r_{NS} + k(\theta_p)\mu_{EW} + \sum_m A_m \cos \Delta \phi_m - B_m \sin \Delta \phi_m
\]

\[
\frac{ds_{EW}}{dt} = r_{EW} - k(\theta_p)\mu_{NS} + \sum_m A_m \sin \Delta \phi_m + B_m \cos \Delta \phi_m
\]

\[
\Delta \phi_m = \phi_{roll} - m\phi_p
\]

\[
Z = C_g \mu_{NS} \cos \phi_r + \delta \phi - \mu_{EW} \sin \phi_r + \delta \phi + b + n
\]
Data Analysis Tools

Trapped Flux Mapping (TFM) → High Frequency Analysis

Scale factor, polhode parameters

2-second Filter
(Geometric Analysis: cross-check) → Low Frequency Analysis

Relativity estimates + a few hundred other parameters

Results evaluation
Trapped Flux Mapping

- Express trapped magnetic potential in spherical harmonics

- Transform TF fixed in body by Euler rotation to inertial frame
  1. About 3-axis by $\Psi_p$ (polhode phase)
  2. About 2-axis by $\theta_p$ (polhode angle)
  3. About 3-axis by $-\Psi_s$ (“spin” phase)

- Trapped Flux Mapping:
  1. Nonlinear estimation of rotor dynamics, $\Psi_p(t), \theta_p(t), \Psi_s(t)$
  2. Linear fit for coefficients of spherical harmonic, $A_{lm}$, $l$ odd

Rotor body-fixed frame

Trapped magnetic potential
# Trapped Flux Mapping (HF Analysis)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Angular velocity, $\omega$</td>
<td>$10 \text{ nHz} \sim 10^{-10}$</td>
</tr>
<tr>
<td>Polhode phase, $\phi_p$</td>
<td>$\sim 1^\circ$</td>
</tr>
<tr>
<td>Rotor orientation</td>
<td>$\sim 2^\circ$</td>
</tr>
<tr>
<td>Trapped magnetic potential</td>
<td>$\sim 1%$</td>
</tr>
<tr>
<td>Gyroscope scale factor, $C_g^{TF}$</td>
<td>$\sim 10^{-4}$</td>
</tr>
</tbody>
</table>

Path of spin axis in gyro body

Trapped magnetic potential

Gyro 1

Relative $C_g$ variations

Time (hours)
The 2-second Filter (LF Analysis)

- LF SQUID data sampled every 2 sec over 1 yr (×4)
- Nonlinear simultaneous estimation of
  - Relativistic drift, scale factor, torque coeffs., telescope params., …
- Batch least-squares fit (Bayesian)
  - Iterative linearization & linear least squares fit
  - Sigma point algorithm for Jacobian computation
    - Robust convergence & unbiased uniform drift estimates
Choosing Baseline Number of Parameters

Relevant Sub-models

- Low frequency scale factor variations
- Trapped flux part of scale factor
- Low freq. misalignment torque coefficient
- Polhode harmonics of torque coefficient
- Telescope scale factor

⇒ Criteria: increase number of terms until $\Delta r_{NS}, \Delta r_{WE} \leq 0.5\sigma$
Parameters & Data Segments & Data Points

- **6 data segments \( \times 4 \) gyros analyzed independently**
  - Consistency of 24 gyro-segments verifies model accuracy
- **6 segments analyzed together for each gyro**
  - Most precise result due to nonlineararities
- **Linear combination of 4 gyros**
  - Final experiment result

### Number of parameters & days of data & data points per gyro

<table>
<thead>
<tr>
<th>No. parameters</th>
<th>Gyro 1</th>
<th>Gyro 2</th>
<th>Gyro 3</th>
<th>Gyro 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. days of data</td>
<td>200.7</td>
<td>219.7</td>
<td>188.8</td>
<td>241.7</td>
</tr>
<tr>
<td>No. 2 s data points</td>
<td>( 8.7 \times 10^6 )</td>
<td>( 9.5 \times 10^6 )</td>
<td>( 8.2 \times 10^6 )</td>
<td>( 10.4 \times 10^6 )</td>
</tr>
</tbody>
</table>
Individual Gyro & Joint Results & GR Prediction

All ellipses are 95% confidence
Cross-checks

1. Consistency of analysis approaches
2. Segment-to-segment consistency
3. Gyro-to-gyro consistency
4. Measurement of Guide Star bending by the Sun
5. Others …
Roll-pohlode Resonance Torque

Electrostatic model predicts Euler spiral motion when harmonic of polhode = roll frequency

Unique behavior clearly seen in “raw” data
Misalignment torque

Geometric method is torque model independent

⇒ Consistent with 2-second Filter with explicit model

From 2-sec Filter

From Geometric method (model independent)
Gyro 4 Per-segment Results without SRE Model
Gyro 4 Per-segment Results with SRE Model
Segment-to-Segment Consistency

Gyro 1

Gyro 2

Gyro 3

Gyro 4
Consistency Among Analysis Approaches

Geometric method less model independent

- Uses geometry to eliminate misalignment torque from data
- Reduced precision relative to 2-second Filter

![Graph showing consistency among analysis approaches. The graph compares Gyro 4, segments 5, 6, 9, 10, with Geometric and 2-sec Filter methods.](image)
Gravitational Deflection of Starlight

- As a cross-check, Geometric method inverted to estimate guide star deflection by the Sun
- Modified gyro rate equations

\[
\frac{ds_{NS}}{dt} = R_{NS} + k \mu_{WE} + \lambda \frac{d}{dt} \phi_{NS}(t)
\]

\[
\frac{ds_{WE}}{dt} = R_{WE} - k \mu_{NS} + \lambda \frac{d}{dt} \phi_{WE}(t)
\]

- GR predicted deflection: 21.7 mas
- GP-B estimate: 21 ± 7 mas
Credibility of the Result

1. Models based on the physics of the experiment
2. Clear separability of relativity from classical effects
3. Parameter sensitivity
   - Result **NOT** from any particular choice of parameters
4. Verification through consistency of results
   - Gyro-to-gyro
   - Segment-to-segment
5. Agreement between separate approaches
   - For both intermediate parameters and relativity estimates

<table>
<thead>
<tr>
<th>GP-B result</th>
<th>-6,601.8 ± 18.3</th>
<th>-37.2 ± 7.2</th>
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<tr>
<td>GR Prediction</td>
<td>-6,606 ± 0.1</td>
<td>-39.2 ± 0.1</td>
</tr>
</tbody>
</table>
Lessons Learned

A. Large gaps and no disturbances to TM

B. The more data the better

C. COMPLEX SPACE EXPERIMENTS DO WORK
Thank you for your attention
UV Charge Management Concept

- Rotor charge controlled via UV excited electrons
- Charge rates ~ 0.1 pC/day
- Continuous measurement at the 0.1 pC level
- Control requirement: 15 pC

Discharge of Gyro1 after levitation

- 450 pC (on levitation)
- 70 pC/hour discharge
- 100 pC
- 0 pC
UV Charge Management Results

1 pC ≡ 1 mV

$C_{\text{gyro}} = 1 \text{ nF}$